Experiment design for batch-to-batch model-based learning control

Marco Forgione[1], Xavier Bombois[1], Paul Van den Hof[2]

[1]Delft University of Technology
Delft Center for Systems and Control

[2]Eindhoven University of Technology
Department of Electrical Engineering

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Motivations

For model-based control,
trade-off between modeling (identification) and control effort.

- The control performance depends on the quality of the model.
- The quality of a model depends on the experimental data.
- Experiments can be expensive (time, materials, performance degradation).

Research Question:
"Can we design experiments in such a way that the overall performance is optimized?"

NOTE:
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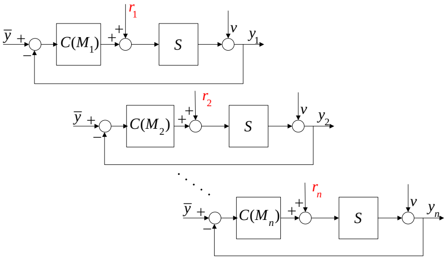
The Framework

Linear, SISO time-invariant system operated in closed-loop over n consecutive intervals (batches).

- After a batch, identification and controller re-design.
- Excitation signal r_k in each batch.



Excitation r_k has a dual effect. Worsens performance during the batch k, but can improve performance for batch $k + 1$.

"Design the excitation signals in order to optimize the performance over n batches."

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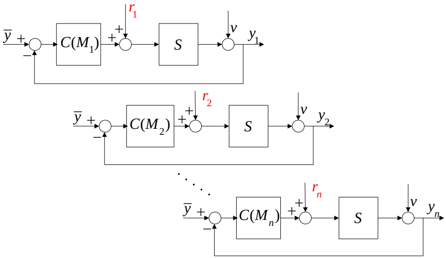

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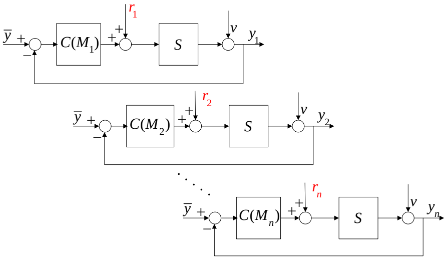

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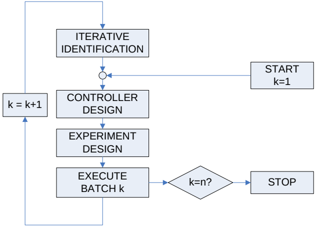
- Real system $\mathcal{S}$: $y = G_o(q)u + H_o(q)e$
  in a model structure $M(\theta)$, i.e. $\mathcal{S} = \mathcal{M}(\theta_o)$.
- $\mathcal{M}(\theta)$ regular, initial estimate $\hat{\theta}_1 \sim \mathcal{N}(\theta_o, R_1^{-1})$ available

Before a batch:

- Identification
- Controller design
- Experiment design



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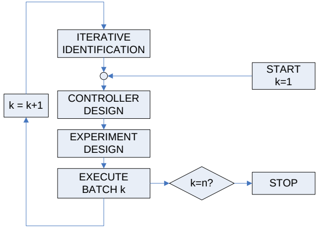
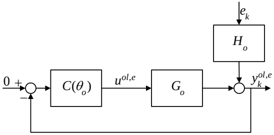


Optimal Loop



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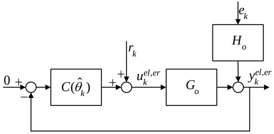

Iterative Identification

Using a Bayesian identification scheme. When the batch k is executed

- Data (y_k, u_k) are collected.
- Previous estimate $\hat{\theta}_k \sim \mathcal{N}(\theta_o, R_k^{-1})$ is available.

The updated MAP parameter estimate $\hat{\theta}_{k+1}$ is computed as

$$\hat{\theta}_{k+1} = \arg\min_{\theta} \frac{1}{\sigma_e^2} \|\epsilon\|_2^2 + \left\|\theta - \hat{\theta}_k\right\|_{R_k}^2$$

The parameter $\hat{\theta}_{k+1} \sim \mathcal{N}(\theta_o, R_{k+1}^{-1})$ with

$$R_{k+1} = R_k + I_k.$$

I_k is the information matrix relative to the experiment k.
I_k is a linear function of the spectrum of excitation signal r_k.

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Controller Design

Based on the parameter $\hat{\theta}_k$, the controller $C_k = C(\hat{\theta}_k)$ is determined
Different controller design strategies $C(\cdot)$ possible...

Here we use an $\mathcal{H}_2$ criterion (mixed sensitivities).
Minimize weighted sum of input/output power.

$$C(\hat{\theta}_k) = \arg\min_K \left\| \begin{matrix} \frac{H(\hat{\theta}_k)}{1+KG(\hat{\theta}_k)} \\ \frac{\sqrt{\gamma}KH(\hat{\theta}_k)}{1+KG(\hat{\theta}_k)} \end{matrix} \right\|_{\mathcal{H}_2}^2 .$$

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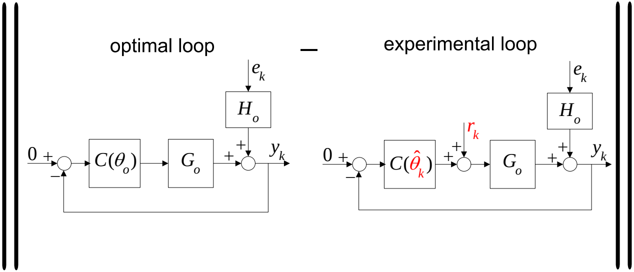
Overview

Define the total cost for a single batch as

$\mathcal{T}_k \triangleq$



Excitation signals r_k are designed in order to

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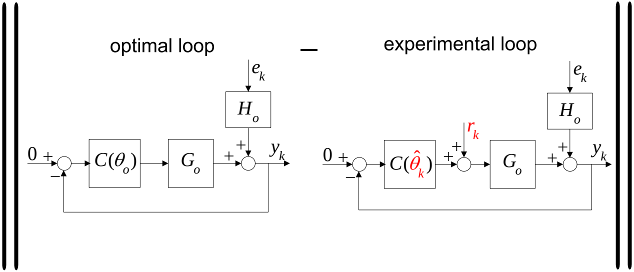

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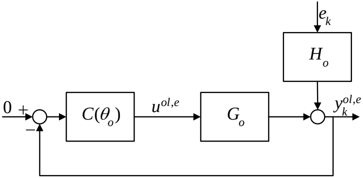
Total Cost, Application Cost & Excitation Cost

Total Cost: power of output difference between the two loops:

$$\mathcal{T}_k \triangleq E[(y_k^{ol,e} - y_k^{el,er})^2].$$

Optimal Loop



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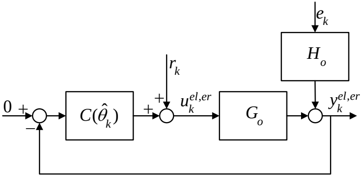

Since $r_k \perp e_k$:

$$\overbrace{E[(y_k^{ol,e} - y_k^{el,er})^2]}^{\text{Total Cost } \mathcal{T}_k} = \overbrace{E[(y_k^{ol,e} - y_k^{el,e})^2]}^{\text{Control Cost } \mathcal{V}_k} + \overbrace{E[(y_k^{el,r})^2]}^{\text{Excitation Cost } \mathcal{E}_k}.$$

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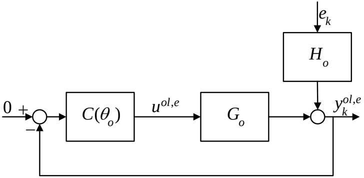

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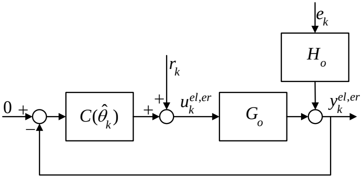

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Objective

- Experiment Design Problem ($k = 1$):
  minimize the summation of the total cost over the n batches

$$\min \sum_{k=1}^{n} \mathcal{T}_k \qquad \text{subject to}$$
$$\mathcal{T}_k \leq \bar{\mathcal{T}}_k, \; k = 1, 2, \ldots, n$$

- Design variables: (spectra of) excitation signals $r_1, r_2, \ldots, r_n$
- $\mathcal{T}_k$ random variables $\Rightarrow$ minimization in a stochastic sense.
  Worst-case with a given probability α.

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Worst-case control cost

From Parseval relation $\mathcal{V}_k = E[(y_k^{ol,e} - y_k^{el,e})^2] =$

$$\mathcal{V}_k(\theta_o, \hat{\theta}_k) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \left| \frac{1}{1 + C(\hat{\theta}_k)G(\theta_o)} - \frac{1}{1 + C(\theta_o)G(\theta_o)} \right|^2 |H|^2 (\theta_o)\sigma_e^2 \, d\omega$$

We approximate $\mathcal{V}_k(\theta_o, \hat{\theta}_k)$ as a quadratic function of θ_o locally around $\hat{\theta}_k$:

$$\mathcal{V}_k(\theta_o, \hat{\theta}_k) \approx \frac{1}{2}(\theta_o - \hat{\theta}_k)^\top V''(\hat{\theta}_k)(\theta_o - \hat{\theta}_k).$$

Since $\theta_o - \hat{\theta}_k \sim \mathcal{N}(0, R_k^{-1})$, using standard ellipsoids we can find the worst-case $\mathcal{V}_k$ with probability α as

$$\mathcal{V}_k^{\mathrm{wc}} = \min_{\lambda_k} \frac{1}{\lambda_k} \qquad \text{s.t } R_k \geq \lambda_k \frac{V'' \chi_\alpha^2(n)}{2} \qquad \text{(convex optimization)}$$

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depends on the decision variables!

Solution based on Randomized Algorithms...
Using the initial estimate $\hat{\theta}_1 \sim \mathcal{N}(\theta_o, R_1^{-1})$:

1. Draw q samples $\tilde{\theta}_s$.
2. Compute $\mathcal{E}_{k,s} = \mathcal{E}_k(\tilde{\theta}_s, \theta_k)$ for $s = 1, \ldots, q$.
3. Extract the empirical maximum $\mathcal{E}_k^{\mathrm{wc}} = \max_s \mathcal{E}_{k,s}$.

The number of samples q can be tuned such that $\mathcal{E}_k^{\mathrm{wc}}$ is the Worst Case Excitation Cost with probability α (randomized algorithms).

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Further Approximation

We will need to evaluate $\mathcal{V}_k^{\mathrm{wc}}, \mathcal{E}_k^{\mathrm{wc}}$ for $k = 1, \ldots, n$ before the execution of the first batch.

- For the Control Cost

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Quantities in red are not known!

Typical chicken & the egg issue of Experiment Design.

They are all replaced with $\hat{\theta}_1$.

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Formulation

Let us define

$$\mathcal{T}_k^{\mathrm{wc}} = \mathcal{V}_k^{\mathrm{wc}} + \mathcal{E}_k^{\mathrm{wc}}, \; k = 1, 2, \ldots, n.$$

The Experiment Design Problem ($k = 1$)

$$\min_{\Phi_{r_1}, \ldots, \Phi_{r_n}} \quad \sum_{k=1}^{n} \mathcal{T}_k^{\mathrm{wc}} \quad \text{subject to}$$

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can be written as a SemiDefinite Program (SDP).

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Receding Horizon Implementation

In order to alleviate the chicken & the egg issue, implementation in Receding Horizon over the batches.

1. Experiment design for batch 1 solved based on $\hat{\theta}_1$. Spectra $(\Phi_{r_1}, \ldots, \Phi_{r_n})$ found.
2. Signal r_1 applied during the batch 1. Batch executed, data (y_1, u_1) collected.
3. Parameter $\hat{\theta}_2$ identified from the data.
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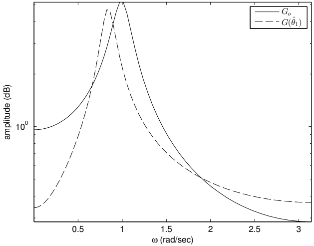
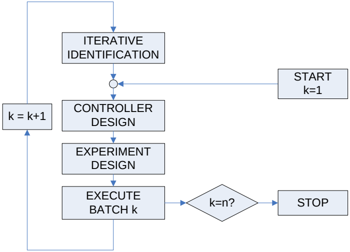
4. Experiment design for batch 1 solved based on $\hat{\theta}_2$. New spectra $(\Phi_{r_2}, \ldots, \Phi_{r_n})$ found.
5. Signal r_2 applied during the batch 2. Data (y_2, u_2) collected.
6. ...

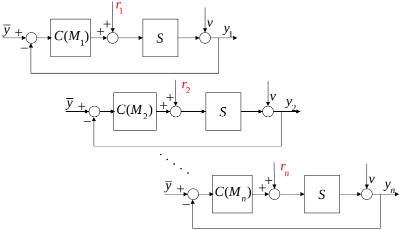
Simulation Case

Second-order system $\mathcal{S}_o$ in a BJ model structure.

- $n = 10$ batches
- of length $N = 200$
- Constraints:
  - $\mathcal{T}_k \leq 0.7$ for $k = 1, \ldots, 6$.
  - $\mathcal{T}_k \leq 0.05$ for $k = 7, \ldots, 10$.

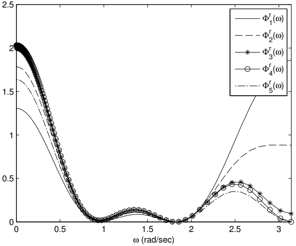




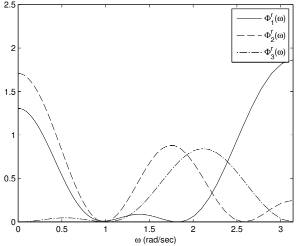


Simulation Case

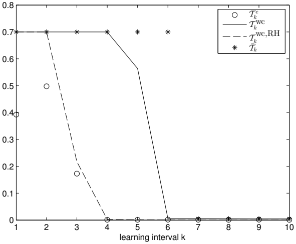
Excitation Spectra



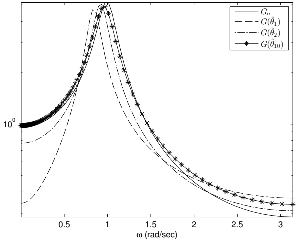
Excitation Spectra RH



Total cost



Bode plot of G



Conclusions

An Experiment Design framework for batch systems

- Optimization of the overall performance.
- No distinction between identification and control batches.
- Excitation only when it pays back.

Open issues

- Approximations to compute the worst-case. Analysis?
- Batch systems are often nonlinear and “short”.
- Initial conditions plays a significative role.

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Thank you.
Questions?